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THE FOURIER LAW AT MACRO AND NANOSCALES

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ABSTRACT

The Fourier law implicitly assumes transient thermal disturbances are carried throughout the solid at an infinite velocity while not defining the carrier mechanism. Paradoxically, the phonon and electron carriers on which the Fourier law is based are limited to acoustic velocities. At the macroscale, the paradox is resolved by the thermal BB photons of QM that carry the Planck energy $E = kT$ of the atoms in the disturbance throughout the solid at the speed of light. BB stands for blackbody and QM for quantum mechanics. The traditional Fourier equation in lattice temperature is expressed in terms of the Planck energy E of the atoms to show infinite carrier velocity is reasonably approximated by BB photons at the speed of light, thereby avoiding the unphysical alternative that absent BB photons the Fourier law is required to rely on thermal disturbances travelling at infinite velocity. Practically, the effect of BB photons on the accuracy of the Fourier solution is insignificant as the BB transient response of the semi-infinite solid is shown identical to that which includes the lag time caused by the speed of light. Fourier's law is not applicable at the nanoscale as by QM the Planck energy of the atom is not available to be carried through the solid by the BB photon.

INTRODUCTION

Since the 19th century, many attempts have been made to explain the infinite velocity of thermal disturbances that underlie the partial differential equation of heat transfer known as Fourier's law. In terms of the lattice temperature T,

$$\rho C \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} + Q \tag{1}$$

where, C is the specific heat, ρ is density, and K is thermal conductivity of the lattice, t is the time and Q describes the source energy per unit of time and volume.

Although Fourier's law has been verified in an uncountable number of heat transfer experiments, the Fourier law itself remains a paradox. Equation (1) states that the temperature rate $\partial T/\partial t$ at a disturbance in a solid is related to the instantaneous lattice temperature T given by the gradient of $\partial T/\partial x$ at some other point perhaps even distantly disposed, thereby suggesting the thermal disturbance travels at an infinite velocity in violation [1] of the theory of relativity. Relativity aside, phonons and electrons on which the Fourier law is based are limited to acoustic velocities.

Indeed, the literature is replete with proposals [2, 3] of modifying the Fourier law to allow infinite velocity or that disturbances are instantaneously known everywhere in a solid. One proposal is the Cattaneo-Vernotte or CV model [1] that assumes the Fourier law is valid at some time after the disturbance occurs. However, the motivation for such proposals is misguided. What the incredible success of the Fourier law in explaining thermal conduction is telling us is the disturbances are indeed instantaneously known everywhere in the solid. Instead of modifying the Fourier law by mathematical trickery to avoid infinite velocity, say by the CV model, we should be looking for a mechanism that reasonably approximates the assumption in the Fourier law that disturbances do indeed travel at an infinite velocity.

Traditionally, the heat carrier [4] in the thermal conduction of solids is the phonon based on theories of Einstein and Debye, although the paradox of an infinite velocity in Fourier's law has never been resolved. Certainly, the phonon at acoustic velocities or the electron in metals does not allow the thermal disturbance of the atom to be instantaneously known everywhere in the solid. A far faster mechanism underlies the incredible success of the Fourier law.

PURPOSE

To propose BB radiation present in all solids – metals and non-metals alike – is the mechanism that validates the Fourier law at the macroscale. Planck's QM allows BB photons to carry the Planck energy of the atom in a thermal disturbance at the speed of light that after a short time delay is known at all other atoms in the solid, a process that reasonably approximates the Fourier law that assumes disturbances travel at an infinite velocity. However, the Fourier law at the nanoscale is no longer applicable as QM precludes the atom from having the Planck energy for the BB photon to carry throughout the solid.

THEORY

QM Restrictions

By the theories of Einstein and Debye, the Fourier law follows [4] classical physics and always allows the atom to have Planck energy $E = kT$ energy or equivalently the capacity to conserve absorbed EM energy by an increase in temperature. In contrast, QM by the Einstein-Hopf relation [5] differs in that the Planck energy E of the atom depends on the wavelength λ of EM confinement. EM stands for electromagnetic. At 300 K, the dispersion of the Planck energy E of the atom with wavelength λ by classical physics and QM is shown in Fig. 1.

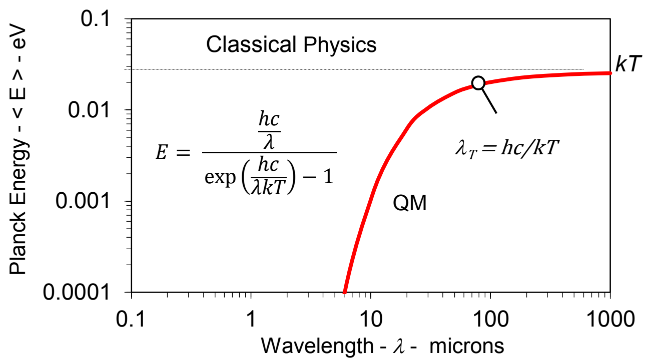

Fig. 1 Heat Capacity of the Atom at 300K
E is Planck energy, h Planck's constant, c speed of light, k Boltzmann's constant, T temperature, and λ wavelength

At the macroscale, the Fourier law is not restricted by QM as the BB photons have the Planck energy $E = hc/\lambda = kT$ of the atom for all $\lambda > \lambda_T$. At 300 K, BB photons have $E = 0.0258$ eV and $\lambda_T = 48$ microns. Otherwise, QM restricts Fourier's law. QM requires BB photons under EM confinement at wavelengths $\lambda < \lambda_T$ to have $E < kT$ while for $\lambda < 6$ microns, $E < 0.0001$ eV. At the nanoscale for $\lambda < 1$ micron, QM requires the Planck energy E of the atom to vanish. Conversely, classical physics allows atoms at the nanoscale to have the same kT energy as for $\lambda > \lambda_T$.

By QM, the Planck energy $E = kT$ of atoms is carried by BB photons throughout the solid at the macroscale, but not at the nanoscale.

EM Confinement and QED

In 1870, Tyndall showed light underwent EM confinement upon being trapped in the surface of a body if the RI of the body is greater than that of the surroundings. TIR stands for total internal reflection and RI for refractive index. Similarly, BB photons created by the temperature of the atom undergo EM confinement at the macro and nanoscale.

At the macroscale, Fourier's law is not modified by QM as structures have TIR confinement wavelengths $\lambda > \lambda_T$. The nanoscale differs. For $\lambda < 1$ micron, nanostructures have high surface to volume ratios, and therefore absorbed EM energy is almost entirely absorbed in their surfaces. Provided the RI of the nanostructure is greater than the surroundings, the surfaces of nanostructures fully participate in the shape of the TIR wave functions. Under TIR confinement, QED induces the absorbed EM energy to undergo spontaneous frequency up-conversion to surface photons having Planck energies at UV levels and beyond. QED stands for quantum electrodynamics. The QED induced photons create excitons (holon and electron pairs) that charge the nanostructure or upon recombination emit EM radiation to the surroundings.

Similarly, structures having $\lambda > 1$ micron also create excitons upon absorbing EM energy, but because the QED induced photons have lower Planck energy than in nanostructures having $\lambda < 1$ micron, far fewer, if any excitons are created. In contrast, at the macroscale having $\lambda > \lambda_T$, absorbed EM energy increases the temperature of the structure. Regardless, TIR confinement at the macro and nanoscale is not permanent, sustaining itself only during the absorption of EM energy, i.e., absent absorbed EM energy, there is no TIR confinement, and therefore QED induced radiation and excitons are not created.

QED relies on complex mathematics as described by Feynman [6] although the underlying physics is simple, i.e., photons of wavelength λ are created by supplying EM energy to a QM box with sides separated by $\lambda/2$, i.e., QED up-converts [7] low-frequency EM energy to the TIR confinement frequency given by the characteristic dimension d of the nano or macrostructure. The Planck energy E of the QED radiation,

$$E = h\upsilon, \quad \upsilon = \frac{c/n}{\lambda}, \quad \lambda = 2d \tag{2}$$

where, n is the RI of the structure. For films and spherical or cylindrical geometries, d is the thickness or diameter.

BB Radiation and Temperature

The BB radiation spectral energy density $U(\lambda,T)$ emitted from the atom at temperature T,

$$U(\lambda,T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{3}$$

where U is given in energy / volume /unit wavelength. The total U_T by integrating over all wavelengths gives the Stefan-Boltzmann equation,

$$U_T(T) = \sigma A T^4 \tag{4}$$

Unlike the nanoscale, thermal conduction at the macroscale characterized by wavelengths $\lambda > 1$ micron is given by the Rayleigh approximation,

$$U(\lambda, T) = \frac{8\pi kT}{\lambda^4} \tag{5}$$

The dispersion of BB radiation at various temperatures T in (3) with wavelength λ is shown in Fig. 2. The Rayleigh relation in is noted for $\lambda > 1$ micron.

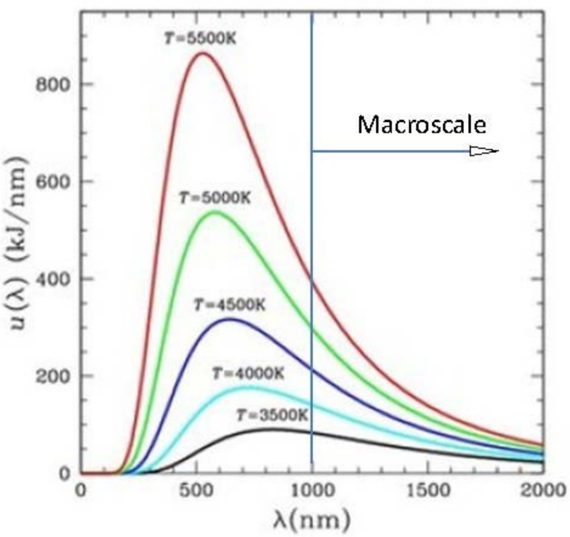

Fig. 2 BB Radiations and Temperature

The Fourier law expressed by Planck energy of Atoms

The Einstein-Hopf relation shows BB photons at the macroscale having wavelengths $\lambda > \lambda_T$ are available to carry Planck energy $E = kT$ throughout the solid. The Fourier law expressed in terms of Planck energy is obtained from (1) by the substitution $T = E/k$,

$$\rho C \frac{\partial E}{\partial t} = K \frac{\partial^2 E}{\partial x^2} + kQ \tag{6}$$

where, Q is independent of T.

In retrospect, the Fourier law may be better understood in terms of BB photons carrying Planck energy (6) instead of by temperature (1) at the speed of light. Indeed, it is more satisfying to interpret the Fourier law in terms of Planck energy E emitted by the atoms. Equation (6) states that the rate $\partial E/\partial t$ at any point in the solid is related to the instantaneous E by the gradient of $\partial E/\partial x$ at some other point, a reasonable statement as BB photons do indeed travel at the speed of light. The Planck energy of the BB photon does depend on the emission temperature of the atom, but it is more satisfying to treat the BB photon as a carrier of Planck energy instead of temperature. In this regard, defining the temperature of the atom by its Planck energy allows the temperature of an atom to be inferred by measuring the Planck energy of the BB emission.

However, for $\lambda < \lambda_T$ the Planck energy $E < kT$ and at the nanoscale E vanishes. Therefore, BB photons can no longer carry the Planck energy of the atom making the Fourier equation invalid at the nanoscale.

Transit Times

At the macroscale, the Planck energy E of the atom is assumed to propagate by BB photons at the speed of light c/n in the solid. In this regard, the BB photons by carrying the Planck energy of the atom may therefore be considered the heat carriers of thermal disturbances at the speed of light. Indeed, the BB photon transit times $\tau = L/c/n$ are very short, e.g., for lengths $L < 1$ m and RI of $n = 1.5$, $\tau < 2$ ns.

Although BB photons move at the speed of light, the thermal response of the solid upon the absorption of the BB photon still depends on the solution of the Fourier equation including local heat sources under appropriate boundary conditions.

SIMULATION

In the BB radiation interpretation, the traditional Fourier law in lattice temperature (1) is simulated for the transient response [8] of a semi-infinite region at temperature T_o subject to a sudden surface temperature T_i. From (1) for $Q = 0$, the Fourier solution for temperature T after time t at distance x is,

$$\Theta = \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \tag{7}$$

where,

$$\Theta = \frac{T - T_o}{T_i - T_o}$$

and $\alpha = K/\rho C$ is the thermal diffusivity. However, the Fourier solution Θ lags the BB solution Θ_{BB} by the speed of light c/n in the solids,

$$\Theta_{BB} = \operatorname{erf}\left(x/2 \sqrt{\alpha\left(t - \frac{x}{c/n}\right)} \right) \tag{8}$$

Assuming a diffusivity $\alpha = 1.7 \times 10^{-6}$ m^2/s and the RI $n = 1.5$, the Fourier Θ and BB Θ_{BB} solutions at $t = 1$ ns are virtually identical as shown in Figs. 3. For $t = 1$ and 2 ns, the difference between Θ_{BB} and Θ solutions is plotted in Fig. 4.

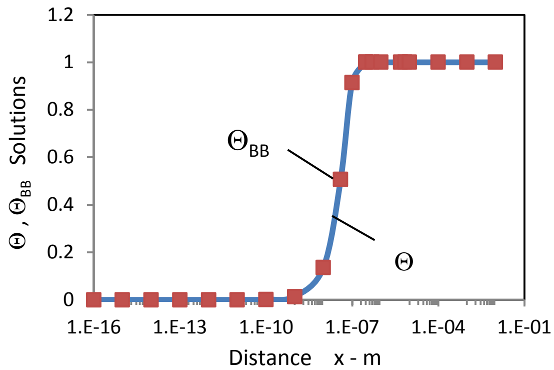

Fig. 3 Transient Solutions Θ , Θ_{BB} at t = 1 ns

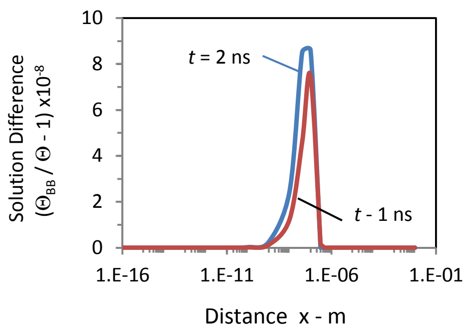

Fig. 4 Solution Difference (Θ_{BB} / Θ - 1) at t = 1 and 2 ns

DISCUSSION

The thermal BB photons that carry the Planck energy of the atoms at the speed of light provide a reasonable approximation to the Fourier law that assumes thermal disturbances travel an infinite velocity in the solid. Although Einstein advanced the concept of quanta beyond that given [9] by Planck, Einstein did not use [10] the word "photon" in his work. Instead, the "photon" originated with Lewis [11] some 20 years later. Of note, Lewis did not consider photons as light or radiant energy but as "the carrier of radiant energy." In this paper, "radiant energy" is taken as the Planck energy of the atom while the "carrier" is the BB photon.

Regardless, Planck's BB radiation is a concept and does not describe a real material, say made up of atoms arranged in a lattice structure. Nothing in Planck's law defines the interaction of BB radiation with matter, and therefore the argument may be made that Planck's law is not sufficiently representative of real materials to justify the validity of Fourier's law.

Fundamentally, Planck's law assumes thermal equilibrium which never occurs in real materials. Indeed, the mechanism of BB radiation interaction with matter became irrelevant once Planck assumed thermal equilibrium.

What then is the BB mechanism in thermal non-equilibrium by which atoms in real materials carry their Planck energy to other atoms throughout the solid?

Irrespective of thermal equilibrium, BB photons are essentially non-interacting with each other, but photon interaction does affect the temperature of matter. The Planck energy of BB photon emission comes from the electrons in the atom even with the solid in thermal non-equilibrium. As the electrons move from one orbit to another within the atom, they produce BB photons. An emission line is produced by an atom in an excited energy state releasing the energy is the form of a BB photon to a lower energy state. Conversely, an absorption line is produced when a BB photon is absorbed by an atom, moving an electron to a higher energy level. Because the energy levels in an atom are fixed, the size of the electron transitions in emission and absorption are the same. Since light is both a photon and a wave, the BB photon carries the Planck energy of every atom throughout the solid as waves travelling at the speed of light.

Regardless of thermal non-equilibrium, BB radiation based on Planck's assumption of BB equilibrium reasonably approximates the response of atoms in real materials.

SUMMARY AND CONCLUSIONS

Macroscale

The Fourier law at the macroscale that implicitly assumes heat carriers travel at an infinite velocity while the phonon and electron are limited to acoustic velocities is resolved by BB photons that carry the Planck energy of the atoms throughout the solid at the speed of light.

The Einstein-Hopf relation at the macroscale for wavelengths $\lambda > \lambda_T$ shows BB photons carry Planck energy $E = kT$ at the temperature T of the atoms in the thermal disturbance throughout the solid. However, once the BB photons are absorbed the response of the solid is not instantaneous. The Fourier equation under local heat sources and boundary conditions still needs to be solved to determine the thermal response as is the case when solving Fourier's equation.

BB radiation carries the Planck energies of the atoms in the disturbance at the speed of light, the process being continuous. Indeed, BB photons are continually being emitted and absorbed throughout the solid with the temperatures constantly being modified by the solution of the Fourier equation for local heat sources and boundary conditions. Of importance is the Planck energy of the atoms in the disturbance is carried at the speed of light consistent with the fact that carriers of infinite velocity underlie the Fourier law.

At the macroscale, the thermal disturbances carried throughout the solid by BB photons do not negate the traditional phonon and electron heat carriers as both are required to update the temperatures of the solid by continuous solutions of the Fourier equation for absorbed Planck energies and other heat sources under local boundary conditions.

The BB photons at the speed of light provide a physical argument in support of the validity of the Fourier law as shown by its incredible success in explaining innumerable heat transfer simulations at the macroscale. There is no need for the CV equation or mathematical trickery to show the Fourier law is valid for phonon and electron heat carriers moving at acoustic velocities instead of infinite velocity. Unequivocally, the Fourier law of heat conduction does not violate the theory of relativity by BB photons travelling at the speed of light.

The Fourier law that assumes thermal disturbances travel at infinite velocity may be explained in terms of the Planck energy of the atom emission carried by BB photons at the speed of light. There is no need to solve typical Fourier thermal conduction problems with BB photons, except in special cases where the time lag in the thermal response is important.

Simulations show the traditional surface lattice temperatures in a semi-infinite solid by the Fourier law are virtually identical to those that lag BB photons by the speed of light.

Nanoscale

By QM, the Fourier law is not applicable at the nanoscale because QM precludes the atom from having the Planck energy to allow being carried by BB photons through the solid.

QM by the Einstein-Hopf relation precludes the atom under TIR confinement at the nanoscale from having the heat capacity to conserve absorbed EM energy by an increase in temperature. Instead, conservation proceeds by the creation of excitons from QED induced EM radiation produced in the atoms under the momentary TIR confinement.

The QED induced excitons charge the nanostructure by holons while the paired electrons escape to the surroundings, or the holons upon recombination with electrons emit EM radiation to the surroundings.

QED radiation produced at the speed of light in the solid is far faster than phonons at acoustic velocities can respond thereby effectively negating thermal conduction by the Fourier law at the nanoscale. Unlike the macroscale, the Fourier law by BB photon carriers fails at the nanoscale because QM requires the Planck energy of the atom to vanish.

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